MASS TRANSFER DURING FILTRATION IN A POROUS MEDIUM WITH A SOLUBLE SKELETON

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An exact solution is obtained for the problem of the washing of a porous half-space in a diffusion-free arrangement. An investigation is made of the characteristic features of mass transfer during the filtration of an active liquid.

The interaction of an active solution with a porous medium plays an important part in various problems of hydrotechnology (filtration in saline soils) and geotechnology (the recovery of minerals by extracting their solutions) [1-3]. The increase of the porosity occurring during such interactions is physically equivalent to the removal of the dissolved material, and in general, this can significantly influence the structure of the solution of the corresponding boundary-value problem.

The characteristic features of mass transfer during the filtration of a solution can be studied through the example of the leaching of a half-space with a constant flow rate in a diffusion-free arrangement. In this case, the mass transfer in the active zone ($0 < N < N_1$, $x < ut/m_1$) can be described by the following equations

$$\frac{\partial (mc+N)}{\partial t} + u\frac{\partial c}{\partial x} = 0, \tag{1}$$

$$\rho m + N = \rho m_1 + N_1 = \text{const}, \ \frac{\partial N}{\partial t} = \beta (c - c_0). \tag{2}$$

The initial and boundary conditions have the form:

$$N(x,\ 0) = N_1,\ c(x,\ 0) = c_0,\ 0 \leqslant x < \infty;\ c(0,\ t) = 0,\ t > 0, \tag{3}$$

$$u = \text{const},\ c_0 = \text{const}.$$

Similar problems have been considered for limiting situations in [3, 4]. The exact solution of the problem (1)-(3) is sought, and it is shown that under the corresponding conditions this problem gives rise to the solutions obtained in [3, 4]. It is noted first that as a result of the condition (2),

$$\frac{\partial c}{\partial x} = -\frac{\rho}{\beta}\frac{\partial}{\partial t}\left(\frac{\partial m}{\partial x}\right).$$

When this is taken into account, Eq. (1) can be written in the form

$$\frac{\partial}{\partial t}\left\{m(c-\rho) - \frac{u\rho}{\beta}\frac{\partial m}{\partial x}\right\} = 0,$$

giving rise to the first integral

$$m(c-\rho) - \frac{u\rho}{\beta}\frac{\partial m}{\partial x} = \varphi(x). \tag{4}$$

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The function $\phi(x)$ at each point x represents the arrival there of the leading edge of the front at the moment of time $t(x) = m_1 x/u$.

From the condition that the porosity is constant ($m = m_1$) at the leading edge of the front

$$\frac{\partial m}{\partial t} + \frac{u}{m_1}\frac{\partial m}{\partial x} = 0 \tag{5}$$

and from the relationship arising from Eq. (2)

$$\frac{\partial m}{\partial t} = -\frac{\beta}{\rho}(c - c_0) \tag{6}$$

it is found that

$$\varphi(x) = m_1(c_0 - \rho) = \text{const.} \tag{7}$$

By combining Eqs. (4), (6) and (7) the following equation is obtained:

$$\frac{\rho m}{\beta}\frac{\partial m}{\partial t} + \frac{u\rho}{\beta}\frac{\partial m}{\partial x} = (c_0 - \rho)(m - m_1), \tag{8}$$

which describes the variation in porosity in the active zone. The general solution of this equation has the form $C_2 = F(C_1)$, where

$$C_1 = (m - m_1)\cdot\exp\frac{\beta(\rho - c_0)x}{u\rho}, \quad C_2 = ut - m_1 x + \frac{u\rho(m - m_1)}{\beta(\rho - c_0)} \tag{9}$$

are integrals of the characteristic system

$$\frac{\beta dt}{\rho m} = \frac{\beta dx}{u\rho} = \frac{dm}{(c_0 - \rho)(m - m_1)}.$$

The characteristics are defined by the relationship $dx/dt = u/m$. Their slope (u/m) is equal to the rate of advance of the solution. At the leading edge of the active front zone $m = m_1$, $x = x_1(t) = ut/m_1$. The process being investigated can be separated into two stages [3]. The first stage extends until $N(0, t) > 0$ at the inlet. The duration T of this stage and the relationship $m(0, t)$ for $0 \leq t \leq T$ are easily found from Eqs. (2) and (3):

$$T = N_1/(\beta c_0), \quad m(0, t) = m_1 + \frac{\beta}{\rho}c_0 t.$$

Here,

$$C_2(0, t) = \frac{u\rho^2}{\beta c_0(\rho - c_0)}C_1(0, t), \quad F(C) = \frac{u\rho^2}{\beta c_0(\rho - c_0)}C. \tag{10}$$

Thus, in the first stage ($0 \leq t \leq T$, $0 \leq x \leq ut/m_1$) the solution of Eq. (8) taking Eqs. (9) and (10) into account has the form

$$ut - m_1 x + \frac{u\rho(m - m_1)}{\beta(\rho - c_0)} = \frac{u\rho^2(m - m_1)}{\beta c_0(\rho - c_0)}\exp\frac{\beta(\rho - c_0)x}{u\rho}. \tag{11}$$

The corresponding concentration distribution is found from (6):

$$c = \frac{\rho c_0\left[1 - \exp\frac{\beta(c_0 - \rho)x}{u\rho}\right]}{\rho - c_0\exp\frac{\beta(c_0 - \rho)x}{u\rho}}. \tag{12}$$

The second stage of the process begins at the moment $t = T$ when the reserve N of soluble material at the entry is exhausted: $N(0, T) = 0$. At this moment the trailing edge of the front $x = x_2(t)$ of the active zone is formed and begins to move. When $t > T$ the zone $x \in (0, x_2)$ is

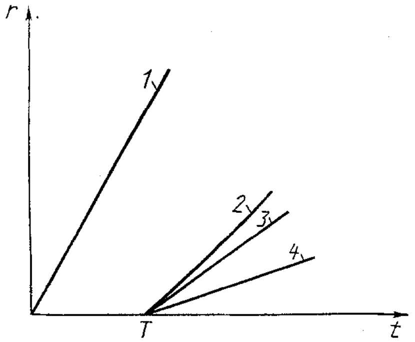

Fig. 1. Form of the active zone of mass transfer: 1) leading edge of the front ($x = ut/m_1$); 2) line (characteristic) of contact of the first and second zones ($x = x_3(t)$); 3) limiting slope of the characteristic ($x = ut/m_2$); 4) trailing edge of the front ($\dot{x}_2 = uc_0/(m_1c_0 + N_1)$).

completely depleted. By analyzing the condition arising from Eq. (1) for the balance of the dissolved material on this front [5] it is found that

$$[mc+N]\dot{x}_2=u[c]. \tag{13}$$

Here the square brackets denote a step change in the quantities contained within them. It should be noted that the parameters m and N are continuous at $x = x_2$, since otherwise as a result of Eq. (2) the front will be stationary over a finite length of time. Hence, instead of Eq. (13) it is possible to write

$$m_2[c]\dot{x}_2=u[c],\ m_2=m_1+\frac{N_1}{p}. \tag{14}$$

If it is assumed here that $[c] \neq 0$, it is found that $\dot{x}_2 = u/m_2$, i.e., the trailing edge of the front is conveyed on a characteristic, which contradicts relationship (11) for $m = m_1$, $x = ut/m_1$. Consequently, this assumption is invalid, $c(x_2(t), t) = 0$ and Eq. (13) does not determine $\dot{x}_2$. The velocity of the trailing edge of the front can be found from the following reasoning. If we have $N(x_2(t), t) = 0$, then

$$\frac{dN}{dt}=\frac{\partial N}{\partial x}\dot{x}_2+\frac{\partial N}{\partial t}=0. \tag{15}$$

On the basis of Eq. (2) it is possible to write

$$N(x,\ t)=N_1+\int\limits_{xm_1/u}^{t}\beta(c-c_0)\,d\tau,$$

where xm_1/u is the moment at which the leading edge of the front passes the point x.

By differentiating with respect to x and substituting the quantity $-\frac{1}{u}\frac{\partial(mc+N)}{\partial t}$ from Eq. (1) in place of $\partial c/\partial x$ in the expression which is obtained, at the moment of time when the trailing edge of the front passes the point x it is found that

$$\frac{\partial N}{\partial x}=\frac{\beta c_0 m_1}{u}+\frac{\beta N_1}{u}.$$

Since $\frac{\partial N}{\partial t}(x_2(t),t)=-\beta c_0$ as a result of E. (2), it is found from Eq. (15) that

$$\dot{x}_2=\frac{uc_0}{m_1c_0+N_1}. \tag{16}$$

For the subsequent discussion it is important to stress that since $N_1 = \rho(m_2-m_1)$ and $\rho > c_0$, then $\dot{x}_2 < u/m_2$. In this connection, in the plane (t, x) a zone is formed (angle BTC) for $t > T$ into which the solution (11) obviously does not extend (see Fig. 1). The structure of the solution of the problem in the second stage is determined by the interaction of two waves: the first of them is generated by the entry ($x = 0$) and is described by relationships (11) and (12), while the second is generated by the trailing edge of the front. The function $C_2 = F(C_1)$ corresponding to it is found by substituting $x = x_2(t)$, $m = m_2$ into the characteristic integrals (9) and eliminating the time t from the expressions which are obtained. As a result of this it will be found that

$$ut - m_1x + \frac{u\rho(m - m_1)}{\beta(\rho - c_0)} = \frac{uN_1\rho}{\beta c_0(\rho - c_0)}\left[\ln\frac{m - m_1}{m_2 - m_1} + 1 + \frac{\beta(\rho - c_0)x}{u\rho}\right], \tag{17}$$

$$c = c_0\frac{N_1 - \rho(m - m_1)}{N_1 - c_0(m - m_1)}. \tag{18}$$

Since from physical considerations the parameters m and N are continuous, as noted earlier, the relationships (11), (12), (17) and (18) completely describe the solution of the problem in the second stage. In particular, by eliminating m from Eqs. (11) and (17) a relationship is obtained for determining the motion $x = x_3(t)$ of the point of contact of the two waves:

$$ut - m_1x_3 + \frac{m_1x_3 - ut}{1 - \frac{\rho}{c_0}\exp\frac{\beta(\rho - c_0)x_3}{u\rho}} = \frac{uN_1\rho}{\beta c_0(\rho - c_0)}\left\{\ln\left[\frac{m_1x_3 - ut}{1 - \frac{\rho}{c_0}\exp\frac{\beta(\rho - c_0)x_3}{u\rho}} \times \frac{\beta(\rho - c_0)}{u\rho(m_2 - m_1)}\right] + 1 + \frac{\beta(\rho - c_0)x_3}{u\rho}\right\}. \tag{19}$$

After this, the step change in the concentration at the point of contact is determined from Eqs. (11), (12) and (18):

$$[c] = \left\{c_0^2\beta\rho(\rho - c_0)^2(ut - m_1x_3)\exp\frac{\beta(\rho - c_0)x_3}{u\rho} + c_0N_1u\rho(\rho - c_0)\left[c_0 - \rho\exp\frac{\beta(\rho - c_0)x_3}{u\rho}\right]\right\}\left\{\left[c_0 - \rho\exp\frac{\beta(\rho - c_0)x_3}{u\rho}\right] \times \left\{N_1u\rho\left[c_0 - \rho\exp\frac{\beta(\rho - c_0)x_3}{u\rho}\right] + c_0^2\beta(\rho - c_0)(ut - m_1x_3)\right\}\right\}^{-1}. \tag{20}$$

The exact solution which has been obtained reduces to the solution of N. N. Verigin [3] under the conditions $c_0/\rho \ll 1$ (small solubility), $x_1(t) \to \infty$ (first stage), $x_3(t) \to \infty$ (second stage). The last two conditions mean that the characteristic dimensions of the zone in which the solution of [3] is observed are much smaller than the dimensions of the active zone in the first stage and the size of the zone occupied by the wave from the trailing edge of the front in the second stage.

The solution given in [4] is obtained from Eqs. (17)-(20) for values of c_0/ρ, $[c](x_3)/c_0$, and $(m_2 - m_1)/m_1$ which are much smaller than unity. Physically this corresponds to a process of washing out a small quantity of a salt of low solubility over a large time of observation. In this case, Eq. (18) reduces to the well known relationship of Zel'dovich [6].

NOTATION

x, coordinate; t, time; m, porosity; c, mass of dissolved material per unit volume of solution; N, mass of soluble material per unit volume of the porous medium as a whole; u, filtration velocity; ρ, density of the soluble material; c_0, equilibrium concentration; β, dissolution rate constant; $\phi(x)$, first integral; C_1, C_2, integrals of characteristic system; m_1, N_1, porosity and mass of soluble material per unit volume of medium as a whole at the initial moment of time $t = 0$; x_2, coordinate giving position of the trailing edge of the front; $\dot{x}_2$, velocity of the trailing edge of the front; x_3, point of contact of the two waves of the solution.

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TIME DEPENDENCE OF THE HEAT-TRANSFER COEFFICIENT BETWEEN COMPONENTS OF A COMPOSITE DURING HEAT TRANSFER

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The effect of the thermophysical and geometrical characteristics of the components of a composite on the dynamic behavior and asymptotic value of the coefficient of heat transfer between the layers is studied.

A multitemperature approach [1-3] based on averaging of the temperature fields of each component within an elementary microvolume is being employed increasingly in the calculation of the thermal state of heterogeneous media. In the case of layered and reinforced media this makes it possible to reduce the dimension of the initial heat equations, thus greatly facilitating the solution of the problem. The resulting system of differential equations (the order of the system is equal to the number of components) is closed by introducing a relation between the density of the thermal flux between the components and their average temperatures. In [1] such a relation was obtained from phenomenological linear relations between the thermodynamic forces and fluxes:

$$q_{ij}=\alpha(\hat{T}_i-\hat{T}_j). \tag{1}$$

It is understood that α is an effective characteristic of the thermophysical and geometric parameters of the structure of the composite. The explicit form for α for a layered composite was obtained in [2] and [3], respectively, as

$$\alpha_2=2\sqrt{3}\frac{l_1 l_2\lambda_1\lambda_2}{l_k^2(l_1\lambda_1+l_2\lambda_2)},\quad \alpha_3=\frac{3\lambda_1\lambda_2}{l_1\lambda_2+l_2\lambda_1}. \tag{2}$$

The heat-transfer coefficient α is an integrated characteristic of the rate of heat transfer between the components. The integrated heat-transfer characteristics are generally not constants. It is known [4], e.g., that the effective thermal-conductivity coefficient, which is also an integrated characteristic, depends on time. By analogy we can assume that α will be a function of time in layered (reinforced) media.

We examine this by considering the model problem of propagation of heat in a two-layer composite with a regular structure (a representative cross section of the material is shown in Fig. 1) under boundary conditions of the second kind. On the assumption that the thermophysical characteristics of the components do not depend on the temperature, we can write the following for an isolated elementary cross section:

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